Lesson-21

Applications of the Theory of Production

Production

The creation of any good or service that has an economic value either to consumers or to other producers is called the production. Production analysis focuses on the efficient use of inputs to create outputs. The process involves all the activities associated with providing goods and services.

Managerial Questions

Managerial questions are as follows:

1. Whether to produce or shut down?
2. How much to produce?
3. What input combination to use?
4. What type of technology to use?

Examples

- Physical processing or manufacturing of material goods
- Production of transportation services
- Production of legal advice
- Production of education
- Production of invention (R&D)
- Production of bank loans

Production Function

The production function relates the output of a firm to the amount of inputs, especially capital and labor.
It can also be defined as a schedule (table, equation) showing the maximum amount of output that can be produced from any specified set of inputs, given the existing technology or "state of the art."

In short, the production function is a catalog of various possibilities of output.

$$Q = f\,(X,Y) \text{ or } (K,L)$$

It is important to keep in mind that the production function describes technology, not economic behavior. A firm may maximize its profits given its production function but generally takes the production function as a given element of that problem. In specialized long-run models, a firm may choose its capital investments to choose among various production technologies.

Economic Efficiency

The production function incorporates the technically efficient method of production. Here, the latest technological processes are used. When the economists use production functions, they assume that the maximum level of output is obtained from any given combination of inputs. It means, they assume that production is technically efficient.

When producers are faced with input prices, the problem is not technical but economic efficiency. How to produce a given amount of output at the lowest possible cost? To be economically efficient, a producer should determine the combination of inputs that solves this problem.

What is technical inefficiency? If, for example, an alternative process can produce the same amount of output using less of one or more inputs and the same amount of all others, the first process is technically inefficient.

If, however, the second process uses less of some inputs but more of others, the economically efficient method of producing a given level of output depends on the prices of the inputs. One might cost less but actually be less technically efficient.

Classifying Inputs

1. **Fixed Input**

   A fixed input is one that is required in the production process. The amount of the fixed input employed is constant over a given period of time regardless of the quantity of output produced.

2. **Variable Input**

   It is the one whose quantity employed in the production process is varied, depending on the desired quantity of output to be produced.

Time Frames

1. **Short-run**

   It is a period of time in which one or more of the inputs are fixed.

2. **Very Short-run**

   It is a period of time in which all resources are fixed.

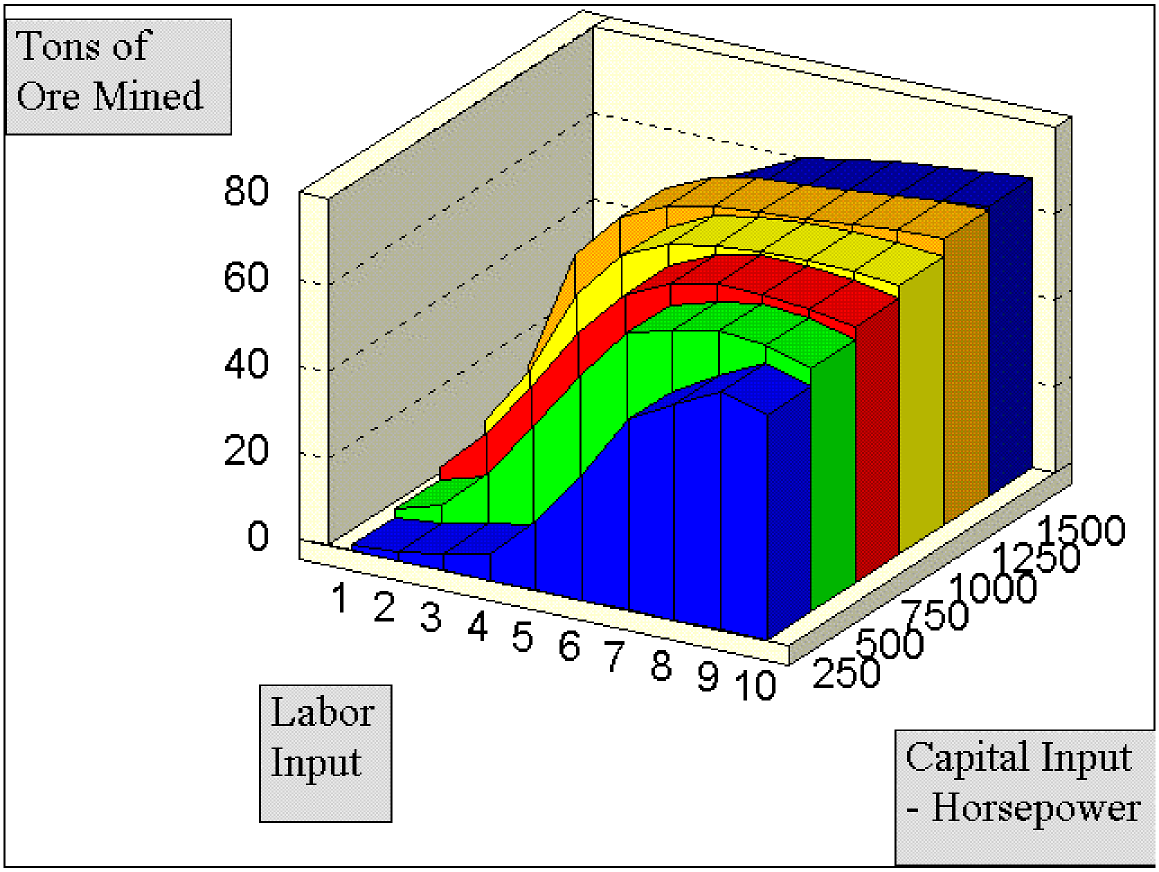
3. Long-run

It is a time period which is long enough so that all resources can be varied.

Example of Ore Mining

Output = tons or ore mined

Labor	Capital (horsepower)							
	250	500	750	1,000	1,250	1,500	1,750	2,000
1	1	3	6	10	16	16	16	13
2	2	6	10	24	29	29	44	44
3	4	16	29	44	55	55	55	50
4	6	29	44	55	58	60	60	55
5	16	43	55	60	61	62	62	60
6	29	55	60	62	63	63	63	62
7	44	58	62	63	64	64	64	64
8	50	60	62	63	64	65	65	65
9	55	59	61	63	64	65	66	66
10	52	56	59	62	64	65	66	67



Labor	Output	MP $\Delta Q \div \Delta X$	AP $(Q \div X)$	Elasticity $MP \div AP$
0	0			
1	6	+6	6	1.0
2	16	+10	8	1.25
3	29	+13	9.67	1.34
4	44	+15	11	1.36
5	55	+11	11	1.0
6	60	+5	10	.50
7	62	+2	8.86	.23
8	62	0	7.75	0.0
9	61	-1	6.78	-.15
10	59	-2	5.90	-.34

Returns to Scale

It is the relation between output and variation in all inputs taken together.

Returns to Factor

It is the relation between output and variation in only one of the inputs employed.

Total Product

It is the total output that results from employing a specific quantity of resources in a production system.

Marginal Product

It is the incremental change in total output that can be produced by the use of one more unit of the variable input in the production process. It can also be defined as the change in output associated with a unit change in one input factor, holding the other inputs constant.

ΔQ-- Change brought about by a change in ΔX units of the variable input and Y remains fixed

$$MP_X = \frac{\Delta Q}{\Delta X}$$

Or, in continuous terms:

$$MP_X = \frac{\partial Q}{\partial X}$$

Average Product

$$AP_X = \frac{Q}{X}$$

It is the ratio of the total output to the amount of the variable input used in producing the output.

Production Elasticity

The percentage change in output results from a given percentage change in the amount of the variable input X employed in the production process is called production elasticity. In this case, Y remains constant. It can also be defined as the percentage change in output associated with a one percent change in all inputs.

$$E_X = \frac{\%\Delta Q}{\%\Delta X}$$

$$= \frac{\Delta Q/Q}{\Delta X/X}$$

By rearranging terms:

$$= \frac{\Delta Q/\Delta X}{Q/X}$$

$$= \boxed{\frac{MP_X}{AP_X}}$$

Law of Diminishing Marginal Returns

It is a principle stating that as more and more of a variable input is combined with a fixed input in short-run production, the marginal product of the variable input eventually declines. This is the economic principle underlying the analysis of short-run production for a firm. Among a host of other things, it offers an explanation for the upward-sloping market supply curve. The law of diminishing marginal returns helps us to understand supply. The law of supply and the upward-sloping supply curve indicate that a firm needs to receive higher prices to produce and sell larger quantities. This observation is easily verified by reviewing the slope of the marginal product curve.

As the number of units of the variable input increases with other inputs held constant, there exists a point beyond which the marginal product of the variable input declines.

It should be noted that it is not a mathematical theorem. It is empirical assertion.

The concepts of total and marginal product and the Law of Diminishing Returns to a factor are important to identify efficient as opposed to inefficient input combinations.

Determining the Optimal Use of the Variable Input

With one of the inputs (Y) fixed in the short-run, a producer should determine the optimal quantity of the variable input (X) to employ in the production process. This is a study of Marginal Revenue Product (MRP) and Marginal Factor Cost (MFC). One can say that this is a study about the role of revenue and cost in the production system.

1. MRP

It is the change in total revenue resulting from a unit change in a variable input. In this case, all other inputs remain unchanged. Marginal revenue product is found by dividing the change in total revenue by the change in the variable input. This is also termed as the value of the marginal product. Marginal revenue product is a key component for understanding the demand for productive inputs, i.e. factor demand.

It can be defined as the amount that an additional unit of the variable input adds to the total revenue. It can also be defined as the economic value of a marginal unit of a particular input factor when used in the production of a specific product.

$$\boxed{MRP = \frac{\Delta TR}{\Delta X}}$$

Where ΔTR is the change in total revenue associated with the given change (ΔX) in the variable input.

MRP_X is equal to the marginal product of (MP_X) times the marginal revenue (MR_Q) resulting from the increase in output obtained.

$$\boxed{MRP = MP_X \cdot MR_Q}$$

Example

Units	TP	MP	MR at $5
1	3	3	$15
2	7	4	20
3	10	3	15
4	12	2	10
5	13	1	5

If the addition of one more laborer to a workforce would result in the production of two incremental units of a product than can be sold for $5, the MP of labor is 2 and its MRP is $10 (2 x $5).

2. Marginal Factor Cost (MFC)

MFC is the amount that an additional unit of the variable input adds to the total cost.

$$\boxed{MFC = \frac{\Delta TC}{\Delta X}}$$

Optimal Input Level

Given the marginal revenue product and marginal factor cost, one can compute the optimal amount of the variable input for using it in the production process.

An economic activity should be expanded as long as the marginal benefits exceed the marginal costs. The optimal point occurs at the point where the marginal benefits are equal to the marginal costs.

$$MRP_X = MFC_X$$

Single Input System

Profit maximization requires production at a level where the marginal revenue is equal to the marginal cost. Because the only variable in the system is input L, the marginal cost of production is as follows:

$$\boxed{MC = \frac{\Delta TC}{\Delta Output}}$$

$$MC = \frac{P_L}{MP_L}$$

Since marginal revenue should be equal to the marginal cost at the profit-maximizing level, MR_Q can be substituted for MC_Q

$$MR_Q = \frac{P_L}{MP_L}$$

Solving for P_L yields:

$$P_L = MR_Q \times MP_L$$

or

$$P_L = MRP_L$$

A profit-maximizing firm always employs an input upto the point where its marginal revenue product equals its cost.

It should be noted that if $MRP_L > P_L$, labor usage should be expanded. And if $MRP_L < P_L$, labor usage should be cutback.

Refer to the ore-mining example:

1. The firm can employ as much labor as it needs by paying workers $50 per period (the labor market is considered perfectly competitive).

2. The firm can sell all the ore it can produce at a price of $10 per ton.

MRP = MFC = $50

3. In a case of less than six workers, MRP > MFC and the addition of more workers will increase revenue. Beyond six, the opposite is true.

Labor	Total Prod.	MP of	TR or P •	MR	MRP	MFC

Input	Tons of Ore	Labor	Q	$\frac{\Delta TR}{\Delta Q}$	MP•MR	
0	0		0			
1	6	6	60	10	60	50
2	16	10	160	10	100	50
3	29	13	290	10	130	50
4	44	15	440	10	150	50
5	55	11	550	10	110	50
6	60	5	600	10	50	50
7	62	2	620	10	20	50
8	62	0	620	10	0	50

The Production Function with Two Variable Inputs

In the ore-mining example, assume that both capital and labor are now variable.

1. Production Isoquant

A production isoquant is either a geometric curve or an algebraic function representing all the various combinations of the two inputs that can be used in producing a given level of output. It can also be defined as a curve (a locus of points) showing all possible combinations of inputs physically capable of producing a given fixed level of output.

2. Marginal Rate of Technical Substitution

It is the rate at which one input may be substituted for another input in the production process. It can also be defined as the rate at which one input is substituted for another along an isoquant.

The rate of change of one variable with respect to another variable is given by the slope of a curve relating the two variables. Thus, the rate of change of input Y with respect to X, i.e. the rate at which Y may be substituted for X in the production process is given by the slope of the curve relating Y to X. This is the slope of the isoquant.

Since the slope is negative and one wishes to express the substitution rate as a positive quantity, a negative sign is attached to the slope.

$$\text{MRTS} = \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{\Delta Y}{\Delta X}$$

For example, in the example of ore mining, moving from 3 to 4 workers yields the MRTS of 250 (horsepower).

$$\text{MRTS} = -\frac{750\text{-}500}{3\text{-}4} = 250$$

For every unit of labor, added 250 horsepower may be discharged without changing the total output. It should be noted that one can show the MRTS equal to the ratio of the marginal products of X and Y.

$$\Delta Y = \frac{\Delta Q}{MP_Y}$$

And

$$\Delta X = \frac{\Delta Q}{MP_X}$$

Substituting these in above yields,

$$MRTS = \frac{MP_X}{MP_Y}$$

The Optimal Combination of Inputs

A firm should make two input choice decisions which are as follows:

1. Choose the input combination that yields the maximum level of output possible with a given level of expenditure.

2. Choose the input combination that leads to the lowest cost of producing a given level of output.

This occurs when, in any constrained optimization problem, one chooses the level of each activity whereby the marginal benefits from the last unit of each activity per dollar cost of the activity are equal. This is known as the equimarginal criterion.

$$\frac{MP_X}{C_X} = \frac{MP_Y}{C_Y}$$

-or-

$$\frac{MP_X}{P_X} = \frac{MP_Y}{P_Y}$$

The optimal combination of inputs in either the cost-minimization or output-maximization problem is a function of the relative prices of the inputs. One can also say that a policy that is cost-effective should arrange things so that the marginal value of emissions is equalized across all sources. This is known as the “equimarginal criterion.”

Changes in Input Prices

Assume that a firm is taking out its production process with the most cost minimizing combination of labor and capital. This is an efficient operation. As we know that:

$$\frac{MP_X}{C_X} = \frac{MP_Y}{C_Y}$$

Suppose, the price of input X rises while the price of input Y is unchanged and the original combination of inputs MP_X and MP_Y are unchanged. At the original combinations, the increase in C_X makes:

$$\frac{MP_X}{C_X} < \frac{MP_Y}{C_Y}$$

Substitution Effect

If a firm wishes to produce the same level of output, it will increase Y and decrease X as it moves along the isoquant.

If the input-price ratio changes, a firm moves ahead toward the input that becomes relatively less expensive and away form the input that becomes relatively more expensive. In the case of labor and capital, if wages/interest increases or decreases, K/L increases or decreases at each level of output. This change in the K/L ratio is called the substitution effect.

Isoquant and Isocost Combinations

Optimal input proportions can be found graphically for a two-input, single-output system by adding a budget line or isocost curve (a line of constant costs) in the diagram of production isoquants.

Each point on a budget line represents some combination of inputs (X and Y) whose cost equals constant expenditure.

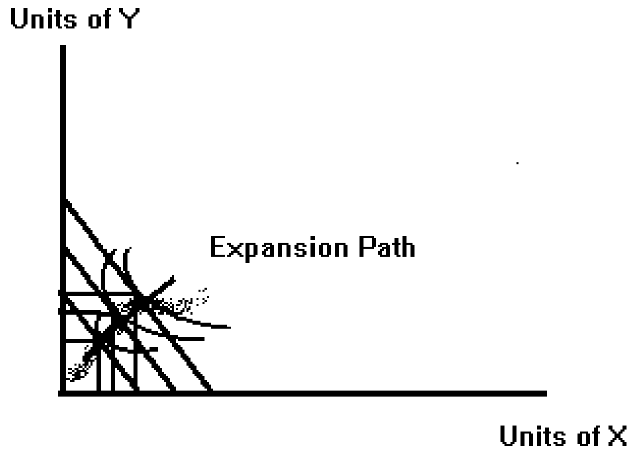


Figure 4.1

The expansion path is the optimal input combination for increasing output. It should be noted that the proportion in which the inputs are combined need not to be same for all levels of output. The expansion path shows how factor proportions change when output changes with the factor-price ratio held constant.

The Decision Making Principle

To minimize the cost (expenditure) of producing a given level of output with fixed input prices, a producer should combine inputs in such quantities that the marginal rate of technical substitution of capital and labor is equal to the input ratio, i.e. the price of labor to the price of capital.

Returns to Scale

A production theory also offers a means for analyzing the effects on output of changes in the scale of production. An increase in the scale of production consists of a simultaneous proportionate increase in all the inputs used in the production process. The proportionate increase in the output of the production process that results from the given proportionate increase in all the inputs is defined as the physical returns to scale.

$$\varepsilon Q = \frac{\%\Delta Q}{\%\Delta X} = \frac{\partial Q}{\partial X} \bullet \frac{X}{Q}$$

Where

$\varepsilon Q > 1$ increasing
$\varepsilon Q = 1$ constant
$\varepsilon Q < 1$ decreasing

1. Increasing-- In this case, output goes up proportionately more than the increase in input usage.
2. Constant-- In this case, output goes up by the same proportion as the increase in input usage.
3. Decreasing-- In this case, output goes up proportionately less than the increase in input usage.

Estimation of Production Function

One of the more common approaches utilizes the Cobb Douglas production function method.

$$Q = \alpha L^{\beta_1} K^{\beta_2}$$

Where

1. Both inputs are required to create output
2. MRTS will diminish as required by production theory

Logarithmic Specification

$$\mathrm{Ln}\, Q = \mathrm{Ln}\, \alpha + \beta_1 \mathrm{LnL} + \beta_2 \mathrm{LnK}$$

Elasticity of Production

$$E_L = \frac{MP_L}{AP_L}$$

Where

$$MP_L = \alpha^{\beta_1} L^{\beta_1 - 1} K^{\beta_2} \quad \text{and,}$$

$$APL = \frac{\alpha L^{\beta_1} K^{\beta_2}}{L} = \alpha L^{\beta_1 - 1} K^{\beta_2}$$

Thus,

$$E_L = \frac{\alpha \beta_1 L^{\beta_1 - 1} K^{\beta_2}}{\alpha L^{\beta_1 - 1} K^{\beta_2}} = \beta_1$$

The exponents- returns to scale

1. Increasing-- $\beta_1 + \beta_2 > 1$
2. Constant-- $\beta_1 + \beta_2 = 1$
3. Decreasing-- $\beta_1 + \beta_2 < 1$

Example

$$Q = 1.01\ {}^{0.75}\ K^{0.25}$$

Q-- An index of physical volume of manufacturing
L-- An index of the average number of employed wage earners only (i.e. salaried employees, officials and working proprietors were excluded)
K-- An index of the value of plants, buildings, tools and machinery reduced to dollars of constant purchasing power

The sum of the exponents were restricted to one. (Constant returns to scale)

Further Studies by Cobb and Douglas

$$Q = .84\, L^{0.63} K^{0.30}$$

A one percent increase in labor input results in about a 2/3 percent increase in output and a one percent increase in capital input results in approximately a 1/3 percent increase in output.

The sum of the exponents is slightly less than one. It seems to indicate the presence of decreasing returns to scale. However, the sum is not significantly different from 1.0. Hence, it really confirms constant returns to scale.

A Three Variable Model

$$Q = \alpha L_p^{\beta_1} L_n^{\beta_2} K^{\beta_3}$$

Where

Q-- The value added by production plants over 18 industries
L_n -- Non-production work-years
L_p -- Production work-hours
K-- Gross book values of depreciable and depletable assets

Empirical Estimation of a Production Function for a Major League Baseball
by CE. Zech as published in the *American Economist*

In an attempt to quantify the factors that contribute to the team's success, a Cobb-Douglas production function was developed using data from the 26 major league baseball teams in 1977. Output (Q) was measured by team victories. Inputs from five different categories were included in the model which are as follows:

- Hitting-- batting average and power (home runs)
- Running-- stolen base record
- Defense-- fielding percentage and total chances accepted
- Pitching-- earned run average (ERA) and strikeouts-to-walks ratio

- Coaching-- Lifetime won-lost record and number of years spent managing in the major leagues
- Dummy-- NL = 0, AL = 1

Variable	Eq. 1	Eq. 2	Eq. 3	Eq. 4
Constant	.017	.018	.010	.008
Dummy	-.002	-.003	.004	.003
B avg.	2.017	1.986	1.969	1.927
Homeruns	.229	.299	.208	.215
Stolen Bases	.119	.120	.110	.112
Strikeouts/Walks	.343	.355	.324	.334
TotField Chances	1.235	1.200		
Field %			5.62	5.96
Mngr.W/L		-.003		-.004
Mngr. Years	-.004		-.004	
R^2	.789	.790	.773	.774

Findings

1. Hitting average contributes almost six times as much as pitching to a team's success. <u>Contradict traditional wisdom?</u>
2. Homeruns contribute about twice as much as stolen bases to a team's success.
3. Coaching skills are not significant in any of the regression equations.
4. Defensive skills are not significant in any of the regression equations.
5. The sum of the statistically significant variables in each of the four equations range from 2.588 to 2.709. Because these are all greater than 1.0, the baseball production functions exhibit increasing returns to scale.